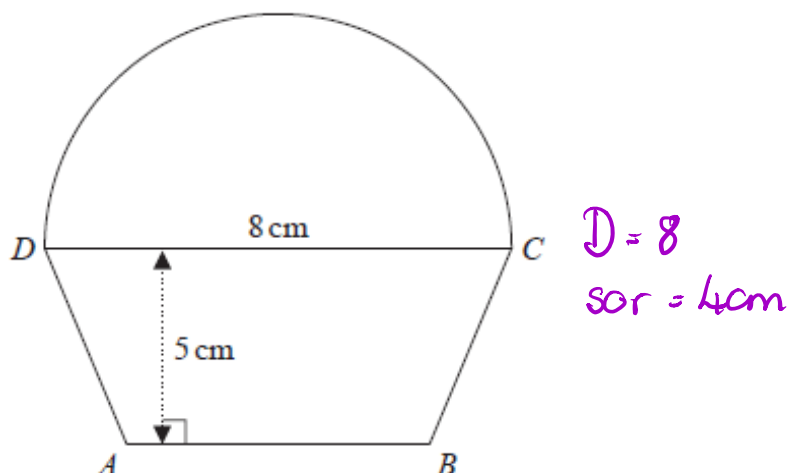


# 1MA1 Higher themed papers: Area and Perimeter - Arcs Sectors Circles

- 1 The diagram shows a shape made from a trapezium  $ABCD$  and a semicircle with diameter  $DC$ .



$DC = 8\text{ cm}$ .

The shape has area  $64\text{ cm}^2$

The height of the trapezium is  $5\text{ cm}$ .

Work out the length of  $AB$ .

Give your answer correct to 1 decimal place.

$$64 = \frac{1}{2} \pi \times 4^2 + \frac{1}{2} (8 + AB) \times 5$$

$$\frac{2(64 - 8\pi)}{5} - 8 = AB$$

$$AB = 7.54690\dots$$

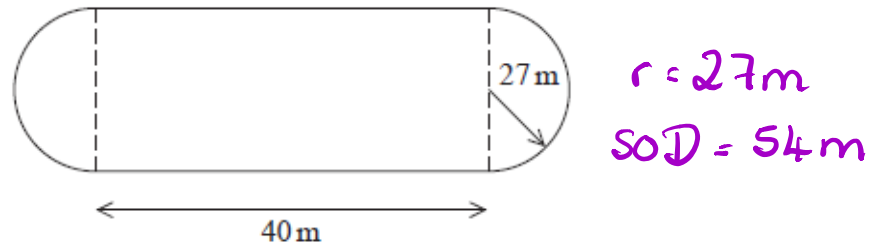
7.5

..... cm

(Total for Question 1 is 5 marks)

## 1MA1 Higher themed papers: Area and Perimeter - Arcs Sectors Circles

- 2 The diagram shows a cycle track.



The track has two straight sides each of length 40 m.  
Each end of the track is a semicircle of radius 27 m.

The diameter of each wheel of Ian's bike is 590 mm.  
Ian is going to ride his bike around the track once.

$$590\text{ mm} = 0.59\text{ m}$$

Calculate how many complete revolutions each wheel of his bike will make.

$$\begin{aligned}\text{One lap of the track} &\Rightarrow \pi \times 54 + 40 + 40 \\ &= 249.6460033\end{aligned}$$

$$\begin{aligned}\text{One revolution of the wheel} &\Rightarrow \pi \times 0.49 \\ &= 0.49\pi\end{aligned}$$

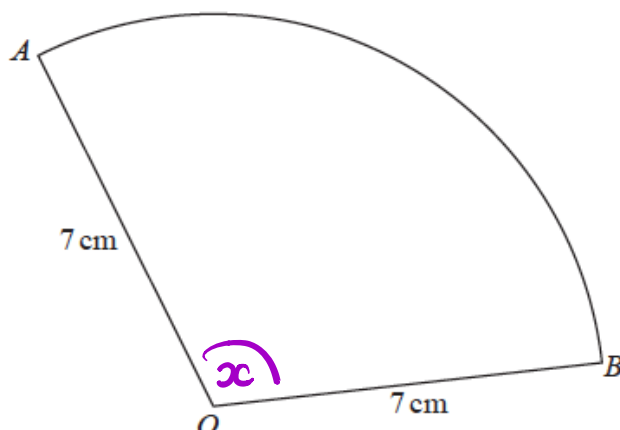
$$\begin{aligned}249.646... \div 0.49\pi \\ = 162.173... \text{ revolutions in one lap of the track}\end{aligned}$$

162

(Total for Question 2 is 5 marks)

### 1MA1 Higher themed papers: Area and Perimeter - Arcs Sectors Circles

- 3  $OAB$  is a sector of a circle with centre  $O$  and radius 7 cm.



The area of the sector is  $40 \text{ cm}^2$

Calculate the perimeter of the sector.

Give your answer correct to 3 significant figures.

$$\frac{x}{360} \times \pi \times 7^2 = 40$$

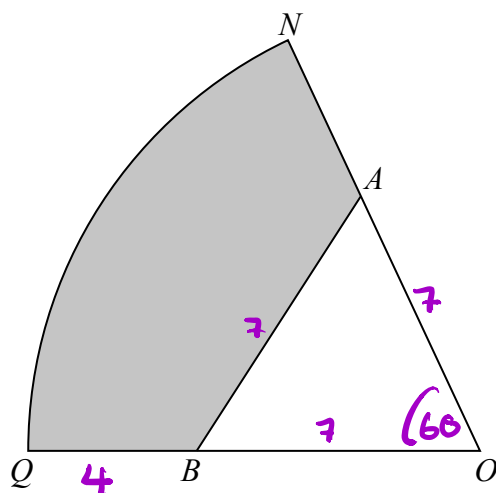
$$x = \frac{40 \times 360}{49\pi}$$
$$= 93.54412982^\circ$$

$$\text{Length of arc} = \frac{93.54}{360} \times \pi \times 14$$
$$= \frac{80}{7} \text{ cm}$$

$$\text{Perimeter} = 7 + 7 + \frac{80}{7} = 25.428571$$

.....  $25.4$  ..... cm  
(Total for Question 3 is 4 marks)

4



$ONQ$  is a sector of a circle with centre  $O$  and radius 11 cm.

$A$  is the point on  $ON$  and  $B$  is the point on  $OQ$  such that  $AOB$  is an equilateral triangle of side 7 cm.

$$\hat{AOB} = 60^\circ$$

Calculate the area of the shaded region as a percentage of the area of the sector  $ONQ$ .  
Give your answer correct to 1 decimal place.

$$\text{area of } NOQ = \frac{60}{360} \times \pi \times 11^2 = \frac{121}{6} \pi$$

$$\text{area of } AOB = \frac{60}{360} \times \pi \times 7^2 = \frac{49}{6} \pi$$

$$\text{area of shaded} = \frac{121}{6} \pi - \frac{49}{6} \pi = 12 \pi$$

$$\% \Rightarrow \frac{12 \pi}{\frac{121}{6} \pi} \times 100 = 59.50413 \dots$$

59.5

.....%

(Total for Question 4 is 5 marks)

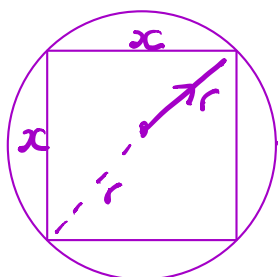
### 1MA1 Higher themed papers: Area and Perimeter - Arcs Sectors Circles

- 5 A square, with sides of length  $x$  cm, is inside a circle.  
Each vertex of the square is on the circumference of the circle.

The area of the circle is  $49 \text{ cm}^2$ .

Work out the value of  $x$ .

Give your answer correct to 3 significant figures.



$$49 = \pi r^2$$

$$r^2 = \frac{49}{\pi}$$

$$r = \sqrt{\frac{49}{\pi}}$$

$$(2r)^2 = x^2 + x^2$$

$$4r^2 = 2x^2$$

$$x^2 = 2r^2$$

$$= 2 \times \frac{49}{\pi}$$

$$x = \sqrt{\frac{98}{\pi}} = 5.585191...$$

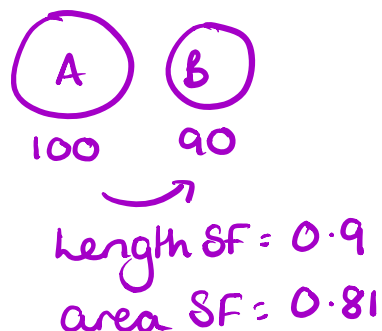
5.59

(Total for Question 5 is 4 marks)

# 1MA1 Higher themed papers: Area and Perimeter - Arcs Sectors Circles

6 The circumference of circle **B** is 90% of the circumference of circle **A**.

(a) Find the ratio of the area of circle **A** to the area of circle **B**.



$$\begin{array}{l} \text{area A} : \text{area B} \\ 100 : 81 \end{array}$$

$$100 : 81$$

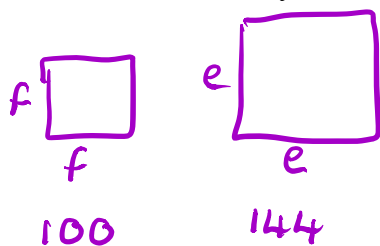
(2)

Square **E** has sides of length  $e$  cm.

Square **F** has sides of length  $f$  cm.

The area of square **E** is 44% greater than the area of square **F**.

(b) Work out the ratio  $e : f$



$$f = 10 \quad e = 12$$

$$\begin{array}{l} e : f \\ 12 : 10 \\ 6 : 5 \end{array}$$

$$6 : 5$$

(2)

(Total for Question 6 is 4 marks)

### 1MA1 Higher themed papers: Area and Perimeter - Arcs Sectors Circles

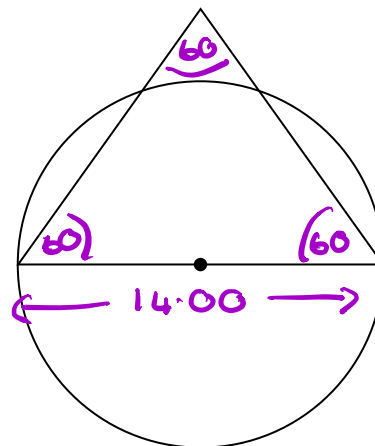
- 7 The diagram shows a circle and an equilateral triangle.

One side of the equilateral triangle is a diameter of the circle.

The circle has a circumference of 44 cm.

Work out the area of the triangle.

Give your answer correct to 3 significant figures.



$$\text{Circumference} = \pi D$$

$$\frac{44}{\pi} = D = 14.00563...$$

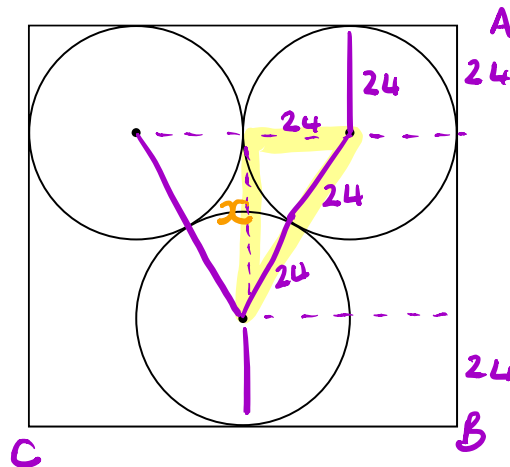
$$\begin{aligned} \text{area of } \Delta &= \frac{1}{2} \times 14.00 \times 14.00 \times \cos 60 \\ &= 49.039452... \end{aligned}$$

$$\underline{\quad\quad\quad 49.0 \quad\quad\quad} \text{cm}^2$$

(Total for Question 7 is 3 marks)

1MA1 Higher themed papers: Area and Perimeter - Arcs Sectors Circles

- 8 The diagram shows 3 identical circles inside a rectangle. Each circle touches the other two circles and the sides of the rectangle, as shown in the diagram.



The radius of each circle is 24 mm.

Work out the area of the rectangle.

Give your answer correct to 3 significant figures.

$$x^2 = 48^2 - 24^2 = 1728$$

$$x = \sqrt{1728} = 24\sqrt{3}$$

$$\text{length of side AB} = 24 + 24 + 24\sqrt{3}$$

$$\text{length of side CB} = 48 + 48 = 96$$

$$\begin{aligned} \text{area} &= \text{AB} \times \text{CB} \\ &= 8598.64506... \end{aligned}$$

8600

..... mm<sup>2</sup>

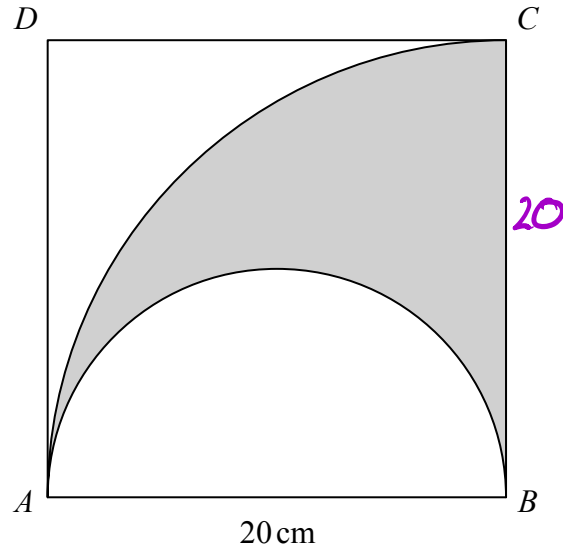
(Total for Question 8 is 4 marks)





9

The diagram shows a square  $ABCD$  with sides of length 20 cm. It also shows a semicircle and an arc of a circle.



$AB$  is the diameter of the semicircle.  
 $AC$  is an arc of a circle with centre  $B$ .

Show that  $\frac{\text{area of shaded region}}{\text{area of square}} = \frac{\pi}{8}$

$$ACB = \frac{1}{4} \pi \times 20^2 = 100\pi$$

$$\text{semicircle } AB = \frac{1}{2} \pi \times 10^2 = 50\pi$$

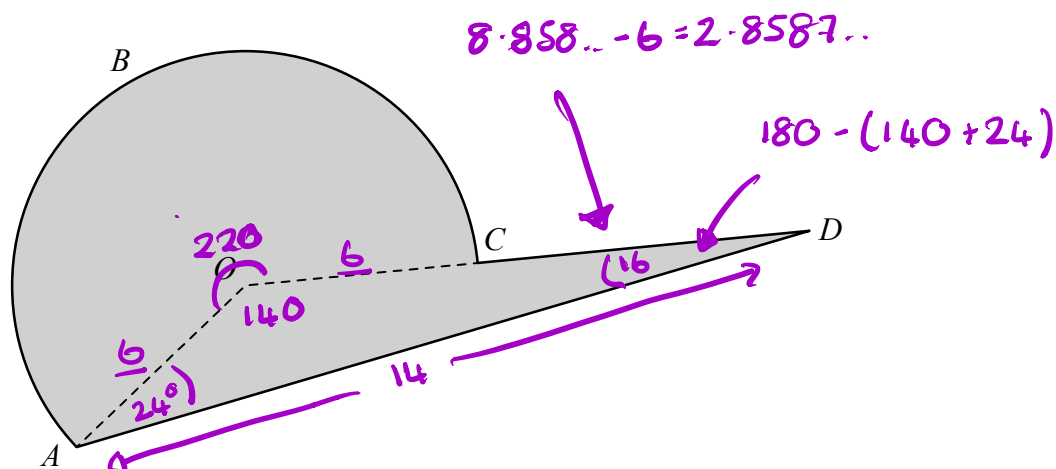
$$\text{shaded area} = 100\pi - 50\pi = 50\pi$$

$$\text{area of square} = 20 \times 20 = 400$$

$$\frac{50\pi}{400} = \frac{25\pi}{200} = \frac{1}{8} \pi \Rightarrow \frac{\pi}{8}$$

(Total for Question 9 is 4 marks)

- 10 Here is a shaded shape  $ABCD$ .



The shape is made from a triangle and a sector of a circle, centre  $O$  and radius 6 cm.  
 $OCD$  is a straight line.

$$AD = 14 \text{ cm}$$

$$\text{Angle } AOD = 140^\circ$$

$$\text{Angle } OAD = 24^\circ$$

Calculate the perimeter of the shape.

Give your answer correct to 3 significant figures.

$$\frac{AO}{\sin 16} = \frac{OD}{\sin 24} = \frac{14}{\sin 140}$$

$$OD = \frac{14}{\sin 140} \times \sin 24 = 8.8587..$$

$$AO = \frac{14}{\sin 140} \times \sin 16 = 6.00341...$$

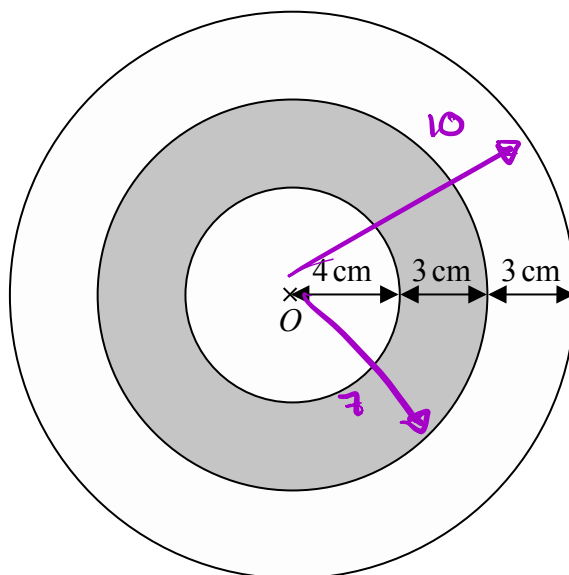
$$ABC = \frac{220}{360} \times \pi \times 6^2 = 23.038..$$

$$\text{Perimeter} = 2.858.. + 23.038 + 14 = 39.896.. \quad \text{39.9 cm}$$

(Total for Question 10 is 5 marks)



- 11 The diagram shows a logo made from three circles.



Each circle has centre  $O$ .

Daisy says that exactly  $\frac{1}{3}$  of the logo is shaded.

Is Daisy correct?

You must show all your working.

$$\pi \times 10^2 = 100\pi$$

$$\pi \times 7^2 = 49\pi$$

$$\pi \times 4^2 = 16\pi$$

$$\text{shaded area} = 49\pi - 16\pi = 33\pi$$

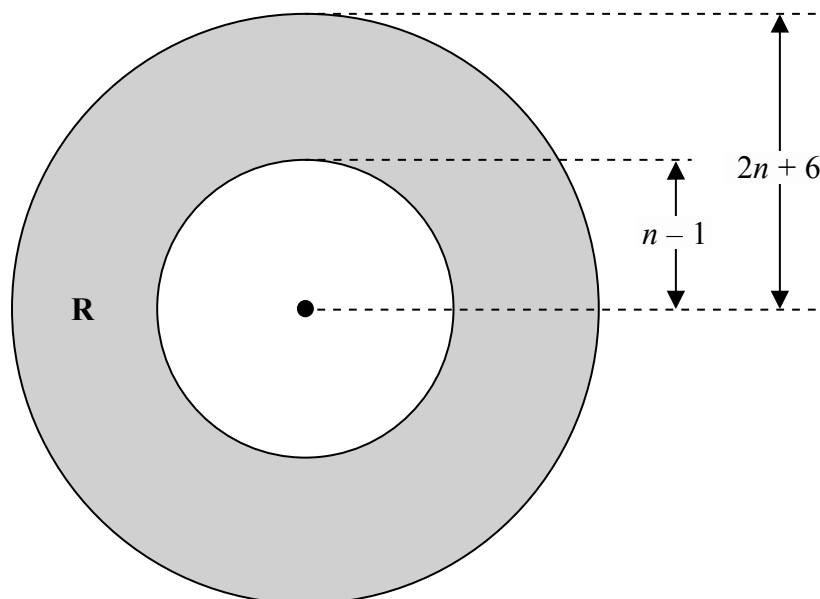
$$\frac{33\pi}{100\pi} = \frac{33}{100}$$

$$\text{Daisy is incorrect } \frac{33}{100} \neq \frac{1}{3}$$

(Total for Question 11 is 4 marks)



- 12 The region **R**, shown shaded in the diagram, is the region between two circles with the same centre.



The outer circle has radius  $(2n + 6)$

The inner circle has radius  $(n - 1)$

All measurements are in centimetres.

The area of **R** is greater than the area of a circle of radius  $(n + 13)$  cm.

$n$  is an integer.

Find the least possible value of  $n$ .

You must show all of your working.

$$\pi(2n+6)^2 - \pi(n-1)^2 > \pi(n+13)^2$$

$$4n^2 + 24n + 36 - (n^2 - 2n + 1) > n^2 + 26n + 169$$

$$4n^2 + 24n + 36 - n^2 + 2n - 1 > n^2 + 26n + 169$$

$$3n^2 + 26n + 35 - n^2 - 26n - 169 > 0$$

$$2n^2 - 134 > 0$$

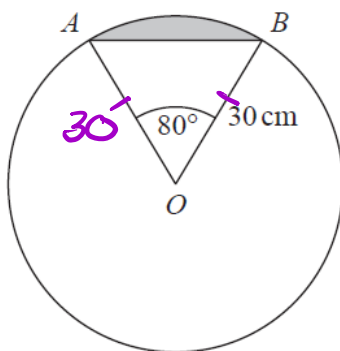
$$n^2 > 67$$

$$8^2 = 64$$

least value of  $n = 9$

(Total for Question 12 is 5 marks)

13



$AB$  is a chord of a circle centre  $O$ .

The radius of the circle is 30 cm.

Angle  $AOB = 80^\circ$

Work out what percentage of the area of the circle is shaded.

area of sector - area of  $\Delta$

$$\frac{80}{360} \times \pi \times 30^2 - \frac{1}{2} 30 \times 30 \times \sin 80$$

$$200\pi - 443.16\dots$$

$$= 188.155\dots$$

$$\% \text{ shaded} = \frac{188.155}{\pi \times 30^2} \times 100$$

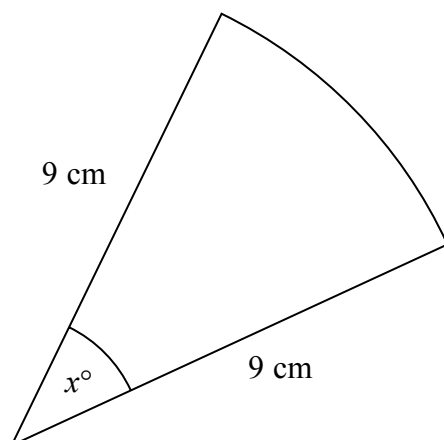
$$= 6.5485\dots$$

..... 6.55 %

(Total for Question 13 is 5 marks)

# 1MA1 Higher themed papers: Area and Perimeter - Arcs Sectors Circles

- 14 The diagram shows a sector of a circle of radius 9 cm.



The sector has a perimeter of 25 cm.

Work out the value of  $x$ .

Give your answer correct to 1 decimal place.

$$\frac{x}{360} \times 18 \times \pi + 9 + 9 = 25$$

$$18\pi x = (25 - 18) \times 360$$

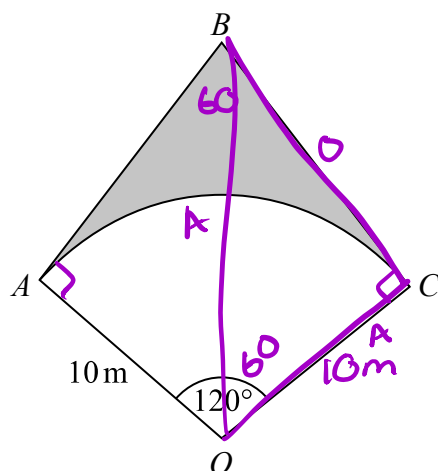
$$x = \frac{2520}{18\pi}$$

$$= 44.563 \dots$$

44.6

(Total for Question 14 is 4 marks)

15



$$BC = 10 \tan 60^\circ = 10\sqrt{3}$$

$$\begin{aligned} \text{area of } \triangle OCB &= \frac{1}{2} \times 10 \times 10\sqrt{3} \\ &= 50\sqrt{3} \end{aligned}$$

$$\text{so area } OACB = 100\sqrt{3}$$

$OAC$  is a sector of a circle, centre  $O$ , radius 10 m.

$BA$  is the tangent to the circle at point  $A$ .

$BC$  is the tangent to the circle at point  $C$ .

Angle  $AOC = 120^\circ$

Calculate the area of the shaded region.

Give your answer correct to 3 significant figures.

$$\text{area of sector } OAC = \frac{120}{360} \times \pi \times 10^2 = \frac{100\pi}{3}$$

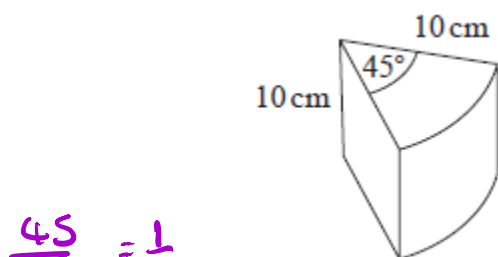
$$\begin{aligned} \text{shaded area} &= 100\sqrt{3} - \frac{100\pi}{3} \\ &= 68.485 \dots \end{aligned}$$

..... $68.5$ ..... $\text{m}^2$

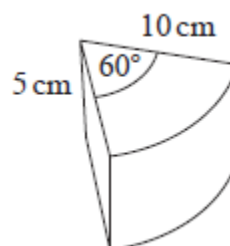
(Total for Question 15 is 5 marks)

# 1MA1 Higher themed papers: Area and Perimeter - Arcs Sectors Circles

- 16 Here are two solid prisms, prism A and prism B.



prism A



prism B

$$\frac{45}{360} = \frac{1}{8}$$

$$\frac{60}{360} = \frac{1}{6}$$

The cross section of prism A is a sector, with angle  $45^\circ$ , of a circle of radius 10 cm. The prism has a depth of 10 cm and a mass of  $40\pi$  grams.

The cross section of prism B is a sector, with angle  $60^\circ$ , of a circle of radius 10 cm. The prism has a depth of 5 cm and a mass of  $50\pi$  grams.

Express the difference in the densities of the two prisms as a percentage of the density of prism A.

VOLUMES

$$\begin{aligned} A & \quad \frac{\pi}{8} \times 10^2 \times 10 \\ & = \frac{1000\pi}{8} \end{aligned}$$

$$\begin{aligned} B & \quad \frac{\pi}{6} \times 10^2 \times 5 \\ & = \frac{500\pi}{6} \end{aligned}$$

Density

$$\begin{aligned} A & = 40\pi \div \frac{1000\pi}{8} \\ & = \frac{320}{1000} = 0.32 \end{aligned}$$

$$\begin{aligned} B & = 50\pi \div \frac{500\pi}{6} \\ & = \frac{300}{500} = 0.6 \end{aligned}$$

$$\frac{0.6 - 0.32}{0.32} \times 100$$

87.5

..... %

(Total for Question 16 is 5 marks)

**TOTAL MARKS FOR PAPER: 71**